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USE OF MAGNETIC THIN FILM IN MICROWAVE PARAMETRIC AMPLIFIER

by

Kung-wei Hsiao

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I. INTRODUCTION

Parametric amplifiers have been developed quite successfully in recent years, and their high gain and low noise properties are very interesting. Most parametric devices use time varying capacitance. However, time varying inductance should also yield equivalent results, as pointed out by Manley and Rowe (1). Suhl (2) proposed to use the ferrite as the time-varying or nonlinear element which couples the pump energy into the signal field to realize amplification. In Suhl's parametric amplifier, the uniform precession of the magnetization vector of a ferrite sample is induced by a local oscillator of frequency f. This uniform precession of the magnetization vector interacts with the signal field of frequency f_1, which then generates a component magnetization of frequency f_2. If a proper microwave structure is used, this magnetization will generate a magnetic field of frequency f_2. Again this field will interact with the uniform precession magnetization. Because of the relation $f_1 + f_2 = f$, this interaction will produce a magnetization of frequency f_1 which, through the microwave structure, will reinforce the original signal field. Hence a regenerative type of amplification or oscillation can be achieved, provided a proper pump field is used. However, as this pump field has to be in resonance with the uniform precession magnetization, most of the energy will be absorbed by the ferrite sample as losses.

Therefore this kind of device needs a very high pump power.

The magnetic thin film has certain interesting properties. Due to the geometric shape, it has a very strong demagnetization field normal to the film plane. Therefore, the saturation magnetization has a tendency to lie in the plane of the film. Also due to the uniaxial anisotropy energy, the 80 percent Ni, 20 percent Fe Permalloy ferromagnetic film exhibits the property that its saturation magnetization tends to lie in a certain direction which is often referred to as the "easy" direction of the magnetic film. The hysteresis loop along this direction exhibits a square form, while the B/H curve in the direction perpendicular to the easy direction is a straight line. The slope of this line is effected by the field applied in the easy direction. Hence a nonlinear type of inductor can be formed which can couple pump power into the signal or oscillating tank circuit. A low-frequency parametric amplifier using a Permalloy ferromagnetic film has been shown to work successfully (3). At higher frequency this operation seems to be possible for such devices. The purpose of this investigation was mainly to study the possibility of using Permalloy thin film for such a device in the very high frequency to microwave frequency range.

The primary purpose of this investigation is to study the threshold pump field for such a device. Therefore, the small amplitude motion equation of the magnetization in

Permalloy thin film will give enough accuracy. This equation is given as follows (4):

$$\frac{\mu_o}{\gamma^2 M} \ddot{\phi} + \frac{\alpha}{\gamma} \dot{\phi} + (H_k + h_p)\phi = h_t. \qquad 1$$

Where

γ = Gyromagnetic ratio

M = Saturation magnetization

H_k = Anisotropy field

α = Loss damping coefficient

and h_p is the r-f field applied along the easy direction of the magnetic thin film at a frequency ω_p. ϕ is the angle between the easy axis and the saturation magnetization. If the angle ϕ is sufficiently small, then the magnetization along the transverse axis is $M\phi$. h_t is the r-f field applied in the transverse direction, as shown in Figure 1. If h_t has a frequency of ω_1, it can be shown that ϕ will have a frequency component of $\omega_p - \omega_1$, or one would have a component of magnetization along the transverse direction of frequency $\omega_2 = \omega_p - \omega_1$. Now suppose that this magnetic film is in a certain microwave structure, and that the microwave structure will support the ω_2 mode. Then this magnetization will produce a magnetic field h_{t2}, provided that the pump field h_p is large enough so that the energy coupled into the ω_2 mode equals the losses of the system so that free oscillation can be maintained. If this h_{t2} field is oriented in the transverse direction of the magnetic film, then it will interact with the pump field, and

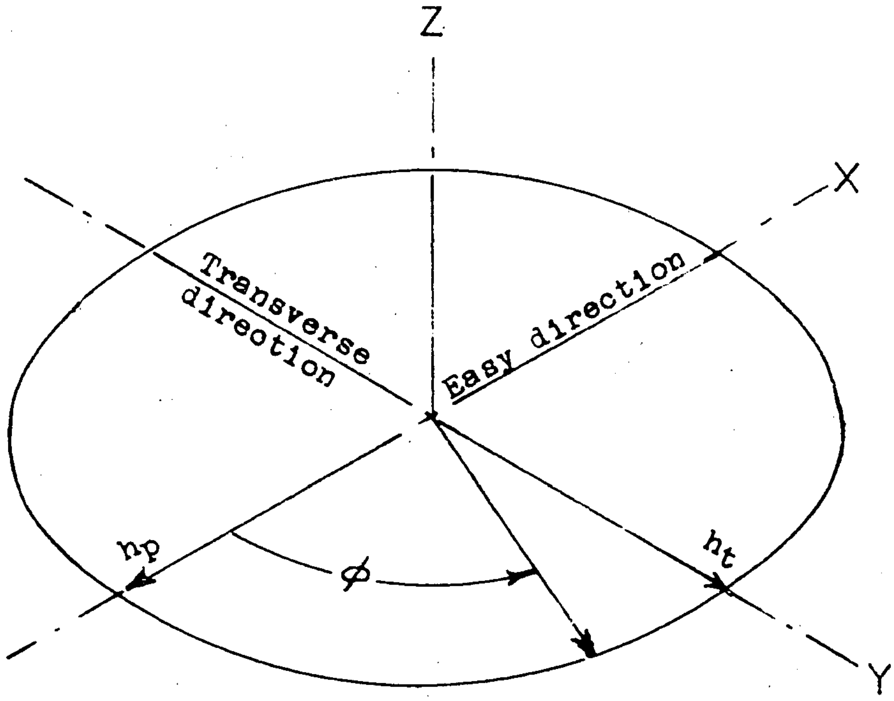

Figure 1. Coordinate system of equation 1

the transverse magnetization of frequency component ω_1 is generated. This magnetization $M_t(\omega_1)$ will produce a magnetic field of frequency ω_1 which will reinforce the signal and hence amplification is realizable. The pump field has to be larger than a certain threshold value. If the pump field is less than this threshold value, the energy pumped into the ω_1 and ω_2 modes will be less than the losses of the system and a damped oscillation will result. By increasing the pump field the energy coupled into the system is also increased until a certain threshold value is reached; then it will break into oscillation. Consequently this threshold pump field intensity is a very important parameter. In order to find the threshold pump field, equation 1 must be solved in terms of h_p and h_t.

II. SOLUTION OF THE SMALL AMPLITUDE GENERAL MOTION EQUATION OF THE MAGNETIZATION OF THIN FILM

Equation 1 is repeated as follows:

$$\frac{\mu_0}{\gamma^2 M}\ddot{\phi} + \frac{\alpha}{\gamma}\dot{\phi} + (H_k + h_p)\phi = h_t. \qquad 1$$

Where h_p is the pump field which can be represented as

$$h_p = 2H_p \cos\omega_p t$$
$$= H_p e^{j\omega_p t} + H_p e^{-j\omega_p t}. \qquad 2$$

Because of the assumption that the microwave structure will support two modes, namely, ω_1 and ω_2, one can assume that the driving function, h_t, is

$$h_t = h_{t1}(\omega_1) + h_{t2}(\omega_2)$$
$$h_{t1}(\omega_1) = H_1 e^{j\omega_1 t} + H_1^* e^{-j\omega_1 t} \qquad 3$$
$$h_{t2}(\omega_2) = H_2 e^{j\omega_2 t} + H_2^* e^{-j\omega_2 t}. \qquad 4$$

With the above assumptions, it can be shown that the solution to equation 1 will have the frequency components, $\omega_1 \pm n\omega_p$ and $\omega_2 \pm n\omega_p$ (5). Equation 1 is essentially Mathieu's equation with a loss term and two driving functions. The general solution takes the following form:

$$\phi = e^{\gamma_1 t}\left[\sum_{-\infty}^{\infty}\phi_{1n}e^{j(\omega_1+n\omega_p)t} + \sum_{-\infty}^{\infty}\phi_{1n}^* e^{-j(\omega_1+n\omega_p)t}\right]$$
$$+ e^{\gamma_2 t}\left[\sum_{-\infty}^{\infty}\phi_{2n}e^{j(\omega_2+n\omega_p)t} + \sum_{-\infty}^{\infty}\phi_{2n}^* e^{-j(\omega_2+n\omega_p)t}\right].$$

The γ can be complex, positive or negative real depending upon the coefficients in equation 1. In the present problem, threshold pump field is being sought that will allow the system to sustain an oscillation. Therefore the steady state solution with $\gamma = 0$ is desired. Thus

$$\phi = \sum_{-\infty}^{\infty} \phi_{1n} e^{j(\omega_1+n\omega_p)t} + \sum_{-\infty}^{\infty} \phi_{1n}^{*} e^{-j(\omega_1+n\omega_p)t} + \sum_{-\infty}^{\infty} \phi_{2n} e^{j(\omega_2+n\omega_p)t} + \sum_{-\infty}^{\infty} \phi_{2n}^{*} e^{-j(\omega_2+n\omega_p)t} . \qquad 5$$

Using the relation $\omega_p = \omega_1+\omega_2$, equation 5 can be changed into the following form:

$$\begin{aligned} \phi = {} & \phi_{10} e^{j\omega_1 t} + \sum_{n=1}^{\infty} \phi_{1n} e^{j[(n+1)\omega_1+n\omega_2]t} \\ & + \sum_{n=1}^{\infty} \phi_{1,-n} e^{-j[(n-1)\omega_1+n\omega_2]t} + \phi_{10}^{*} e^{-j\omega_1 t} \\ & + \sum_{n=1}^{\infty} \phi_{1,n}^{*} e^{-j[(n+1)\omega_1+n\omega_2]t} \\ & + \sum_{n=1}^{\infty} \phi_{1,-n}^{*} e^{j[(n-1)\omega_1+n\omega_2]t} + \phi_{20} e^{j\omega_2 t} \\ & + \sum_{n=1}^{\infty} \phi_{2n} e^{j[(n+1)\omega_2+n\omega_1]t} \\ & + \sum_{n=1}^{\infty} \phi_{2,-n} e^{-j[(n-1)\omega_2+n\omega_1]t} + \phi_{20}^{*} e^{-j\omega_2 t} \\ & + \sum_{n=1}^{\infty} \phi_{2n}^{*} e^{-j[(n+1)\omega_2+n\omega_1]t} \\ & + \sum_{n=1}^{\infty} \phi_{2,-n}^{*} e^{j[(n-1)\omega_2+n\omega_1]t} . \end{aligned}$$

By changing the index of the third, sixth, ninth and twelfth terms of the right side of the equation, some terms can be lumped together. Finally one comes to the following equation by setting m = n-1.

$$
\begin{aligned}
\phi = {} & (\phi_{10}+\phi^*_{2,-1})e^{j\omega_1 t} \\
& + \sum_{n=1}^{\infty}(\phi_{1n}+\phi^*_{2,-m})e^{j[(n+1)\omega_1+n\omega_2]t} \\
& + (\phi^*_{10}+\phi_{2,-1})e^{-j\omega_1 t} \\
& + \sum_{n=1}^{\infty}(\phi^*_{1n}+\phi_{2,-m})e^{-j[(n+1)\omega_1+n\omega_2]t} \\
& + (\phi_{2,0}+\phi^*_{1,-1})e^{j\omega_2 t} \\
& + \sum_{n=1}^{\infty}(\phi_{2n}+\phi^*_{1,-m})e^{j[(n+1)\omega_2+n\omega_1]t} \\
& + (\phi^*_{2,0}+\phi_{1,-1})e^{-j\omega_2 t} \\
& + \sum_{n=1}^{\infty}(\phi^*_{2n}+\phi_{1,-m})e^{-j[(n+1)\omega_2+n\omega_1]t}
\end{aligned}
$$

By reindexing, one has

$$
\begin{aligned}
\phi = {} & \sum_{n=0}^{\infty}\phi_{1n}e^{j[(n+1)\omega_1+n\omega_2]t} + \sum_{n=0}^{\infty}\phi^*_{1n}e^{-j[(n+1)\omega_1+n\omega_2]t} \\
& + \sum_{n=0}^{\infty}\phi_{2n}e^{j[(n+1)\omega_2+n\omega_1]t} \\
& + \sum_{n=0}^{\infty}\phi^*_{2n}e^{-j[(n+1)\omega_2+n\omega_1]t} \\
= {} & \sum_{n=0}^{\infty}\phi_{1n}e^{j(\omega_1+n\omega_p)t} + \sum_{n=0}^{\infty}\phi^*_{1n}e^{-j(\omega_1+n\omega_p)t} \\
& + \sum_{n=0}^{\infty}\phi_{2n}e^{j(\omega_2+n\omega_p)t} + \sum_{n=0}^{\infty}\phi^*_{2n}e^{-j(\omega_2+n\omega_p)t}.
\end{aligned}
$$

Now suppose the pump field is of the form

$$h_p = H_p e^{j\omega_p t} + H_p e^{-j\omega_p t}.$$

Then the mixed-term or the h_p product is

$$\begin{aligned} h_p = & \sum_{n=0}^{\infty} \phi_{1n} H_p [e^{j[\omega_1+(n+1)\omega_p]t} + e^{j[\omega_1+(n-1)\omega_p]t} \\ & + \sum_{n=0}^{\infty} \phi_{1n}^{*} H_p [e^{-j[\omega_1+(n+1)\omega_p]t} \\ & + e^{-j[\omega_1+(n-1)\omega_p]t}] \\ & + \sum_{n=0}^{\infty} \phi_{2n} H_p [e^{j[\omega_2+(n+1)\omega_p]t} + e^{j[\omega_2+(n-1)\omega_p]t} \\ & + \sum_{n=0}^{\infty} \phi_{2n}^{*} H_p [e^{-j[\omega_2+(n+1)\omega_p]t} \\ & + e^{-j[\omega_2+(n-1)\omega_p]t} . \end{aligned} \qquad 7$$

By putting equations 3, 4, 6, and 7 into equation 1, and defining

$$A_{1n} = \frac{H_p}{-(\omega_1+n\omega_p)^2 \frac{\mu_o}{\gamma^2 M} + H_k + j(\omega_1+n\omega_p)\frac{\alpha}{\gamma}}$$

$$A_{2n} = \frac{H_p}{-(\omega_2+n\omega_p)^2 \frac{\mu_o}{\gamma^2 M} + H_k + j(\omega_2+n\omega_p)\frac{\alpha}{\gamma}} \qquad 8$$

$$a_1 = \frac{H_{T1}}{-\omega_1^2 \frac{\mu_o}{\gamma^2 M} + H_k + j\omega_1\frac{\alpha}{\gamma}}$$

$$a_2 = \frac{H_{T2}}{-\omega_2^2 \frac{\mu_o}{\gamma^2 M} + H_k + j\omega_2\frac{\alpha}{\gamma}} \qquad 9$$

one obtains the following simultaneous equations:

$$\Phi_{10} + A_{10}(\Phi_{11}+\overset{*}{\Phi}_{20}) = a_1$$

$$\Phi_{20} + A_{20}(\Phi_{21}+\overset{*}{\Phi}_{10}) = a_2$$

$$\Phi_{11} + A_{11}(\Phi_{12}+\Phi_{10}) = 0$$

$$\Phi_{21} + A_{21}(\Phi_{22}+\Phi_{20}) = 0 \qquad 10$$

$$\vdots$$

$$\Phi_{1n} + A_{1n}(\Phi_{1,n+1}+\Phi_{1,n-1}) = 0$$

$$\Phi_{2n} + A_{2n}(\ _{2,n+1}+\ _{2,n-1}) = 0.$$

By separating Φ_{1n} and Φ_{2n}, and by solving the Φ_{1n} equations first, one has

$$\Phi_{11} + A_{11}\Phi_{12} = -A_{11}\Phi_{10}$$

$$A_{12}\Phi_{11} + \Phi_{12} + A_{12}\Phi_{13} = 0$$

$$\vdots$$

$$A_{1n}\Phi_{1,n-1} + \Phi_{1,n} + A_{1n}\Phi_{1,n+1} = 0.$$

Now, by defining

$$\Delta_{1n} = \begin{vmatrix} 1 & A_{1n} & 0 & 0 & ----- \\ A_{1,n+1} & 1 & A_{1,n+1} & 0 & ----- \\ 0 & A_{1,n+2} & 1 & A_{1,n+2} & ----- \\ 0 & 0 & A_{n+3} & 1 & ----- \\ 0 & & & & \end{vmatrix}$$

it is possible to solve the above simultaneous equations and obtain

$$\Phi_{11} = -A_{11}\Phi_{10} \frac{\Delta_{12}}{\Delta_{11}} .$$

However, from the definition of Δ_{1n}, one gets

$$\Delta_{1n} = \Delta_{1,n+1} - A_{1n}A_{1,n+1}\Delta_{1,n+2}$$

$$\frac{\Delta_{1,n+1}}{\Delta_{1n}} = \frac{1}{1 - A_{1n}A_{1,n+1}\dfrac{\Delta_{1,n+2}}{\Delta_{1,n+1}}} . \qquad 11$$

Using equation 11, one finds

$$\Phi_{11} = -A_{11}\Phi_{10} \frac{1}{1 - A_{11}A_{12}\dfrac{\Delta_{1,3}}{\Delta_{1,2}}}$$

$$= -A_{11}\Phi_{10} \frac{1}{1 - \dfrac{A_{11}A_{12}}{1 - \dfrac{A_{12}A_{13}}{1 - - - -}}} . \qquad 12$$

Since

$$A_{1n} = \frac{H_p}{-(\omega_1+n\omega_p)^2 \dfrac{\mu_o}{\gamma^2 M} + H_k + j(\omega_1+n\omega_p)\dfrac{\alpha}{\gamma}}$$

it is seen that as n becomes large

$$A_{1n} \approx - \frac{H_p}{n^2\omega_p^2 \dfrac{\mu_o}{\gamma^2 M}} .$$

Since H_p is a constant, one can choose an n such that

$$A_{1n}A_{1,n+1} < \alpha_1 \qquad 13$$

where α_1 is a suitably small positive number. When equation 13 is satisfied, one can also say that

$$A_{1(n+p)}A_{1(n+p+1)} < \alpha_1 .$$

Also, by equation 13, one has

$$1 - A_{1n}A_{1,n+1} > 1 - \alpha_1 \qquad 14$$

$$\frac{A_{1n}A_{1,n+1}}{1 - A_{1,n+1}A_{1,n+2}} < \frac{\alpha_1}{1 - \alpha_1} \qquad 15$$

or

$$\frac{A_{1n}A_{1,n+1}}{1 - \cfrac{A_{1,n+1}A_{1,n+2}}{1 - \cfrac{A_{1,n+2}A_{1,n+3}}{1-\text{------}}}} < \frac{\alpha_1}{1 - \cfrac{\alpha_1}{1 - \cfrac{\alpha_1}{1\text{---}}}} . \qquad 16$$

The right side has a limit σ_1

$$= \frac{1}{2}\left(1 - \sqrt{1-4\alpha_1}\right) \qquad 17$$

because one must have

$$\sigma_1 = \frac{\alpha_1}{1 - \alpha_1} .$$

Therefore

$$\Phi_{11} = -A_{11}\Phi_{10}\frac{1}{1 - \cfrac{A_{11}A_{12}}{1 - \cfrac{A_{12}A_{13}}{1\text{-----}\sigma_1}}} . \qquad 18$$

By a similar process one finds

$$\Phi_{21} = -A_{11}\Phi_{20}\frac{1}{1 - \cfrac{A_{21}A_{22}}{1 - \cfrac{A_{22}A_{23}}{1\text{-----}\sigma_2}}} . \qquad 19$$

Now set up

$$\Phi_{11} = -A_{11}\Phi_{10}K_1$$

$$\Phi_{21} = -A_{21}\Phi_{20}K_2 .$$

Then from equation 10, one finds

$$\Phi_{10} + A_{10}(-A_{11}\Phi_{10}K_1) + A_{10}\overset{*}{\Phi}_{20} = a_1 \qquad 20$$

$$\Phi_{20} + A_{20}(-A_{21}\Phi_{20}K_2) + A_{20}\Phi_{10}^{*} = a_2. \qquad 21$$

By solving for Φ_{10} and Φ_{20}, one finds

$$\Phi_{10} = \frac{a_1[1-(A_{20}A_{21}K_2)^{*}] - A_{10}a_2^{*}}{(1-A_{10}A_{11}K_1)(1-(A_{20}A_{21}K_2)^{*}) - A_{10}A_{20}^{*}} \qquad 22$$

$$\Phi_{20} = \frac{a_2[1-(A_{10}A_{11}K_1)^{*}] - A_{20}a_1^{*}}{(1-A_{20}A_{21}K_1)[1-(A_{10}A_{11}K_1)^{*}] - A_{20}A_{10}^{*}}. \qquad 23$$

Therefore the magnetization along the transverse direction is

$$M_{1T} = M(\Phi_{10}e^{j\omega_1 t} + \Phi_{10}^{*}e^{-j\omega_1 t}) \qquad 24$$

$$M_{2T} = M(\Phi_{20}e^{j\omega_2 t} + \Phi_{20}^{*}e^{-j\omega_2 t}). \qquad 25$$

The exact solution of equations 21 and 22 is impossible to find without first knowing the value of H_p. However the value of H_p is a parameter which is being sought. In view of the fact that A_{1n} and A_{2n} are usually very close to a real number with magnitude much smaller than unity, one way to begin the solution is to make the following assumptions:

$$A_{10}A_{11}K_1 \approx 0$$

$$A_{20}A_{21}K_2 \approx 0.$$

By using these assumptions, one would be able to calculate H_p. Once this threshold pump is determined, the coefficient A_{10}, A_{11}, K_1 ...... can be easily computed. Then this value of H_p can be checked. Therefore by the above assumption, one has

$$\phi_{10} = \frac{a_1 - A_{10}a_2^*}{1 - A_{10}A_{20}^*} \qquad 26$$

$$\phi_{20} = \frac{a_2 - A_{20}a_1^*}{1 - A_{10}^*A_{20}} \ . \qquad 27$$

Let

$$H_{T1} = A_1 h_{t1}(\vec{r})$$

$$H_{T2} = A_2 h_{t2}(\vec{r})$$

$$\rho_1 e^{j\theta_1} = H_k - \omega_1^2 \frac{\mu_o}{\gamma^2 M} + j\omega_1 \frac{\alpha}{\gamma}$$

$$\rho_2 e^{-j\theta_2} = H_k - \omega_2^2 \frac{\mu_o}{\gamma^2 M} - j\omega_2 \frac{\alpha}{\gamma}$$

$$\rho e^{j\theta} = \rho_1 e^{j\theta_1} \rho_2 e^{-j\theta_2} - H_p^2$$

then

$$M_{T1} = \frac{MA_1 h_{t1} \rho_2 e^{-j\theta_2} - MH_p A_2^* h_{t2}}{\rho e^{j\theta}} \qquad 28$$

$$M_{T2} = \frac{MA_2 h_{t2} \rho_1 e^{-j\theta_1} - MH_p A_1^* h_{t1}}{\rho e^{-j\theta}} \ . \qquad 29$$

III. RESONANT CAVITY OSCILLATOR BY USE OF MAGNETIC THIN FILM

To begin the solution of this problem one may start by using the general theory of resonant cavities as given by Slater (6). A similar procedure has been used by H. Suhl (2) and Poole (7). Assume that there is a set of orthogonal functions such as

$$\vec{E} = \sum_n b_n(t)\vec{e}(\vec{r}) \qquad 30$$

$$\vec{H} = \sum_n a_n(t)\vec{h}(\vec{r}) \qquad 31$$

where $a_n(t)$ and $b_n(t)$ are functions of time, $\vec{e}(\vec{r})$ and $\vec{h}(\vec{r})$ are functions of space. $\vec{E}$ and $\vec{H}$ are the E-fields and H-fields in the cavity. Also, $\vec{e}(\vec{r})$ and $\vec{h}(\vec{r})$ satisfy the following conditions:

$$\nabla \times \vec{e}_n = K_n\vec{h}_n$$

$$\nabla \times \vec{h}_n = K_n\vec{e}_n$$

$$K_n = \Omega_n/\overline{E_o\mu_o}$$

$$\int_v \vec{e}_n \cdot \vec{e}_m dv = \int_v \vec{h}_n \cdot \vec{h}_m dv = 0 \qquad \text{if } m \neq n.$$

The magnetization vector M exists only in the sample and is zero elsewhere. Therefore

$$\vec{M} = \sum_n \vec{h}_n \frac{\int_{sample} \vec{M} \cdot \vec{h}_n dv}{\int_v h_n^2 dv} .$$

By the above assumption, one finds

$$\nabla \times \vec{E} = \sum_n b_n K_n \vec{h}_n \qquad 32$$

$$\nabla \times \vec{H} = \sum_n a_n K_n \vec{e}_n . \qquad 33$$

Maxwell's equation for this case are

$$\nabla \times \vec{E} = -\mu_o \frac{d\vec{H}}{dt} - \frac{d\vec{M}}{dt} \Big| \text{ sample}$$

$$\nabla \times \vec{H} = E_o \frac{d\vec{E}}{dt} .$$

This relies on the fact that no charge or current source exists in the cavity. From the above equation one finds

$$\sum_n b_n K_n \vec{h}_n = -\mu_o \frac{d}{dt} \sum_n a_n \vec{h}_n - \frac{d}{dt} \sum_n \frac{\int_s \vec{M} \cdot \vec{h}_n dv}{\int_v h_n^2 dv} \vec{h}_n$$

$$\sum_n a_n K_n \vec{e}_n = E_o \frac{d}{dt} \sum_n b_n \vec{e}_n .$$

If the cavity is operating very close to its resonant mode, one can solve the above two equations for the n=1 mode and obtain

$$b_1 k_1 + \mu_o \dot{a}_1 = - \frac{d}{dt} \frac{\int_s \vec{M}_1 \cdot \vec{h}_1 dv}{\int_v h_1^2 dv} \qquad 34$$

$$a_1 k_1 - E_o \dot{b}_1 = 0. \qquad 35$$

In order to see under what condition oscillation is possible, one can make the following assumption:

$$a_1 = A_1 e^{j\omega_1 t + \lambda t} + A_1^* e^{\lambda^* t - j\omega_1 t} . \qquad 36$$

A solution such that λ will be zero or positive real is sought. This really means that the solution for a_1 is such that fields in the system are in oscillation or building up exponentially. By solving equations 34 and 35, one gets

$$\ddot{a}_1 + a_1\Omega_1^2 = -\frac{d^2}{dt^2}\frac{1}{\mu_0}\frac{\int_s \vec{M}_1\cdot\vec{h}_1 dv}{\int_{v_1} h_1^2 dv} \quad . \qquad 37$$

Note that M_1 has the same time varying coefficient as the H fields. Using equation 36, one finds

$$\left[\Omega_1^2 + (j\omega_1+\lambda)^2\right]A_1 = -(j\omega_1+\lambda)^2 \frac{\frac{1}{\mu_0}\int_s \vec{M}_1\cdot\vec{h}_1 dv}{\int_v h_1^2 dv} \quad . \qquad 38$$

By taking the losses of the cavity into account, the resonant frequency of the cavity is changed into a complex quantity

$$\Omega_1 \longrightarrow \Omega_1\left(1 + j\frac{1}{2Q_1}\right)$$

$$\Omega_1^2 \longrightarrow \Omega_1^2 + j\frac{\Omega_1^2}{Q_1} \quad .$$

Assuming that $\lambda << \omega_1$, one has

$$\left[\Omega_1^2 + j\frac{\Omega_1^2}{Q_1} - \omega_1^2 + 2j\omega_1\lambda\right] = -(j\omega_1 + \lambda)^2 \frac{\frac{1}{\mu_0}\int_s \vec{M}_1\cdot\vec{h}_1 dv}{\int_v h_1^2 dv} \quad .$$

39

The integral on the right side of equation 39 can be expanded by using equation 28, with the assumption that h_1 of the cavity is the same field as h_{t1} applied to the film, also h_p is uniform in the film.

$$\frac{\int_s \vec{M}_1\cdot\vec{h}_1 dv}{\int_v h_1^2 dv} = \frac{A_1 M\int_s h_{t1}^2 \rho_2 e^{-j\theta_2} dv}{\left(\int_v h_1^2 dv\right)\rho e^{j\theta}} - \frac{H_p A_2^* M\int_s h_{t1}h_{t2} dv}{\left(\int_v h_1^2 dv\right)\rho e^{j\theta}}$$

40

The first term on the right side of this equation has an A_1 term with a complex coefficient. If it is lumped into the left side of equation 39, its real part merely shifts the frequency, and the imaginary part changes the Q of the cavity. If the cavity is tuned to resonance with ω_1 and the Q term includes the film losses, then one will have

$$j\left(\frac{\omega_1^2}{Q_1} + 2\lambda\omega_1\right)A_1 = (j\omega_1+\lambda)^2 \frac{H_p A_2^* M \int_s h_{t1} h_{t2} dv}{\mu_o \left(\int_v h_1^2 dv\right) \rho e^{j\theta}} . \qquad 41$$

By a similar process, one has

$$j\left(\frac{\omega_2^2}{Q_2} + 2\lambda\omega_2\right)A_2 = (j\omega_2+\lambda)^2 \frac{H_p A_1^* M \int_s h_{t1} h_{t2} dv}{\mu_o \left(\int_v h_2^2 dv\right) \rho e^{-j\theta}} \qquad 42$$

where ω_2 is assumed to be the second mode of the cavity. Now set

$$F_1 = \frac{\int_s h_{t1} h_{t2} dv}{\int_v h_{t1}^2 dv}$$

$$F_2 = \frac{\int_s h_{t1} h_{t2} dv}{\int_v h_{t2}^2 dv} .$$

By neglecting the on the right side of equations 41 and 42, and by taking the conjugate of equation 42, one finds

$$\left(\frac{\omega_1}{Q_1} + 2\lambda\right)A_1 = j\omega_1 \frac{H_p M}{\mu_o \rho} e^{-j\theta} F_1 A_2^* \qquad 43$$

$$\left(\frac{\omega_2}{Q_2} + 2\lambda^*\right)A_2^* = - j\omega_2 \frac{H_p M}{\mu_o \rho} e^{-j\theta} F_2 A_1 . \qquad 44$$

By multiplying these two equations, one obtains

$$\left(\frac{\omega_1}{Q_1} + 2\lambda\right)\left(\frac{\omega_2}{Q_2} + 2\overset{*}{\lambda}\right) = \omega_1\omega_2\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2(\cos 2\theta - j\sin 2\theta). \quad 45$$

In view of equation 45, if $2\theta \neq 0, 2\pi\cdots\cdots$, then the solutions of λ must be complex. Assume that

$$2\lambda = \lambda_1 + j\lambda_2.$$

Then, equating the real part and imaginary of both sides of equation 45, one obtains

$$\lambda_1^2 + \lambda_2^2 + \lambda_1\left(\frac{\omega_1}{Q_1} + \frac{\omega_2}{Q_2}\right) + \frac{\omega_1\omega_2}{Q_1Q_2} = \omega_1\omega_2\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2\cos 2\theta$$

$$\lambda_2\left(\frac{\omega_2}{Q_2} - \frac{\omega_1}{Q_1}\right) = -\omega_1\omega_2\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2\sin 2\theta.$$

Combining these two equations, one finds

$$\lambda_1^2 + \lambda_1\left(\frac{\omega_1}{Q_1} + \frac{\omega_2}{Q_2}\right) + \frac{\omega_1\omega_2}{Q_1Q_2} - \omega_1\omega_2\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2\cos 2\theta + \left[\frac{\omega_1\omega_2\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2\sin 2\theta}{\left(\frac{\omega_2}{Q_2} - \frac{\omega_1}{Q_1}\right)}\right]^2 = 0. \quad 46$$

From equation 46 one sees that in order for the solution of λ_1 to be zero or positive real, the following condition must be satisfied:

$$-\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2\cos 2\theta \geq \frac{1}{Q_1Q_2} + \omega_1\omega_2\left[\frac{\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2\sin 2\theta}{\left(\frac{\omega_2}{Q_2} - \frac{\omega_1}{Q_1}\right)}\right]^2 \quad 47$$

if $\theta = 0$, then

$$\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2 \geq \frac{1}{Q_1Q_2}.$$

Since the second term on the right side of this inequality is always positive, the angle θ must be equal to $n\pi(n=0,1\cdots\cdots)$ in order that $\frac{H_pM}{\mu_o\rho} F_1F_2$ to be minimum. If θ is in the region from $\pm\frac{\pi}{4}$ to $\pm\frac{3}{4}\pi$, the left side becomes negative; then the system is always attenuative.

According to equations 8, 9, and 28

$$\rho e^{j\theta} = (H_k-\omega_1^2\frac{\mu o}{\gamma^2 M} + j\omega_1\frac{\alpha}{\gamma})(H_k-\omega_2^2\frac{\mu o}{\gamma^2 M} - j\omega_2\frac{\alpha}{\gamma}) - H_p^2$$

$$\rho^2 = \left[(H_k-\omega_1^2\frac{\mu o}{\gamma^2 M})(H_k-\omega_2^2\frac{\mu o}{\gamma^2 M}) + \omega_1\omega_2(\frac{\alpha}{\gamma})^2 - H_p^2\right]^2$$

$$+\left[\omega_1 - (H_k-\omega_2^2\frac{\mu o}{\gamma^2 M}) - \omega_2\frac{\alpha}{\gamma}(H_k-\omega_1^2\frac{\mu o}{\gamma^2 M})\right]^2 \qquad 48$$

$$= \tan^{-1}\frac{\omega_1\frac{\alpha}{\gamma}\left[H_k-\omega_2^2\frac{\mu o}{\gamma^2 M}\right] - \omega_2\frac{\alpha}{\gamma}(H_k-\omega_1^2\frac{\mu o}{\gamma^2 M})}{(H_k-\omega_1^2\frac{\mu o}{\gamma^2 M})(H_k-\omega_2^2\frac{\mu o}{\gamma^2 M}) + \omega_1\omega_2(\frac{\alpha}{\gamma})^2 - H_p^2}. \qquad 49$$

Without going any further, one sees that if $\omega_1 = \omega_2$, θ will be equal to $n\pi(n=0,1\cdots\cdots)$.

The resonant frequency of a magnetic film is defined as

$$\omega_r = \gamma\sqrt{\frac{H_kM}{\mu o}}$$

where H_k includes both d-c bias and anisotropy field. For Permalloy, the anisotropy field is about 200 ampere-turns per meter, and

$$\mu_o = 1.26 \times 10^{-6} \text{ weber/amp-turn/meter}$$

$$\gamma = 2.21 \times 10^5 \text{ radians/sec-amp-turn/meter}$$

$\alpha = .015.$

If there is no d-c bias applied, then its resonant frequency is about

$$f_{r\gamma} = \frac{1}{2\pi} \times 2.21 \times 10^5 \sqrt{\frac{200 \times 1}{1.26 \times 10^{-6}}}$$

$$= 450 \times 10^6 \text{ cps.}$$

If both signal frequency and idling frequency are well below this resonant frequency, then $H_k >> \omega_{1,2}^2 \frac{\mu_o}{\gamma^2 M}$, and one has

$$|\rho|^2 \approx (H_k^2 - H_p^2) + \left[\frac{\alpha}{\gamma} H_k(\omega_1 - \omega_2)\right]^2$$

$$\theta \approx \tan^{-1} \frac{H_k \frac{\alpha}{\gamma} (\omega_1 - \omega_2)}{H_k^2 - H_p^2} .$$

Therefore θ will be very close to zero, and the threshold pump field is

$$H_{ps} = \sqrt{\frac{1}{Q_1 Q_2 F_1 F_2}} \frac{\mu_o (H_k^2 - H_p^2)}{M} \qquad 50$$

if

$$F_1 F_2 = \frac{\left(\int_s h_{t1} h_{t2} dv\right)^2}{\int_v h_1^2 dv \int_v h_2^2 dv} \approx \left[\frac{V_{film}}{V_{cavity}}\right]^2 .$$

Therefore H_{ps} becomes

$$H_e \approx \sqrt{\frac{1}{Q_1 Q_2}} \frac{V_{cavity}}{V_{film}} \frac{\mu_o H_k^2}{M}, \text{ if } H_p << H_k. \qquad 51$$

A numerical example follows. Let

$$\omega_1 = 2\pi \times 10^8$$

$$\omega_2 = 2\omega_1$$

$$\frac{\mu_o}{\gamma^2 M} = .25 \times 10^{-16}$$

$$\frac{\alpha}{\gamma} = 6.8 \times 10^{-8}$$

$$\rho e^{j\theta} = 3.35 \times 10^4 - H_p^2 - j9.3 \times 10^2$$

$$\theta = \tan^{-1} \frac{-9.3}{3.35} \times 10^{-2} \approx -1.7 \text{ degree.}$$

Suppose that the cavity has a rectangular shape, and the film thickness is about 2000Å. Then

$$\frac{V_{sample}}{V_{cavity}} \approx 10^{-7}.$$

Let Q_1 and Q_2 be the order of 10^4. Then from equation 51 one finds the threshold pump field to be

$$H_{ps} = \frac{1}{10^4} \times \frac{1}{10^{-7}} \times 1.26 \times 10^{-6} \times (189^2 - H_{ps}^2)$$

$$\approx 37.6 \text{ amp-turn/meter.}$$

For the case of an oscillator, Q_1 of equation 51 is the loaded Q of the cavity; whereas for the amplifier, Q_1 is the unloaded Q of the cavity. From equation 51 one can see that increasing the ratio of the volume of the magnetic film to the volume of the cavity will decrease the threshold pump field. However, the thickness of the film is limited owing to both its inherent property and the skin effect at high frequency. Therefore this volume ratio (or filling factor, as it is sometimes called) is usually very small. This is a disadvantage when the application of the magnetic film is in the high frequency range (8). Secondly, note that H_p is proportional to the square of H_k. Therefore the application of a high d-c bias in the easy direction will increase the pump field. However without sufficient d-c bias, the film tends to break into multiple domains; therefore, bias is almost necessary in order to keep the proper function of the device.

If either signal frequency or idling frequency is near or at the resonant frequency of the magnetic film, then

$$\theta = \tan^{-1} \frac{\omega_1 \frac{\alpha}{\gamma} (H_k - \omega_2^2 \frac{\mu o}{\gamma 2M})}{\omega_1\omega_2 (\frac{\alpha}{\gamma})^2 - H_p^2} \qquad 52$$

and

$$\rho^2 = \left[\omega_1\omega_2 (\frac{\alpha}{\gamma})^2 - H_p^2\right]^2 + \omega_1 \frac{\alpha}{\gamma} (H_k - \omega_2^2 \frac{\mu o}{\gamma 2M}). \qquad 53$$

If ω_2 is either well above or below the resonant frequency of the film, then the angle θ will be in the region $\pm \frac{\pi}{4}$ to $\frac{3}{4}\pi$. From equation 45, λ will always be negative, hence oscillation and amplification is impossible. On the other hand, if $\omega_1 = \omega_2$ then

$$\rho = \omega_1\omega_2 (\frac{\alpha}{\gamma})^2 - H_p^2$$

$$\theta \approx 0.$$

In this case both oscillation and amplification are possible, and the threshold pump field is minimum, and given by

$$H_p = \sqrt{\frac{1}{Q1Q2}} \frac{V_{cavity}}{V_{film}} \frac{\mu o}{M}\left[\omega_1\omega_2 (\frac{\alpha}{\gamma})^2 - H_p^2\right]. \qquad 54$$

But under this condition, the film's lossy term as indicated by equation 40 will be maximum, because the denominator of that expression is close to zero. This will affect both Q_1 and Q_2, and in turn, it will tend to increase the threshold field.

If both modes are operated well above resonant frequency, then

$$\omega_1^2 \frac{\mu_o}{\gamma^2 M} >> H_k$$

$$\omega_2^2 \frac{\mu_o}{\gamma^2 M} >> H_k$$

$$\rho^2 = \left[\omega_1^2\omega_2^2\left(\frac{\mu_o}{\gamma^2 M}\right)^2 + \omega_1\omega_2\left(\frac{\alpha}{\gamma}\right)^2 - H_p^2\right]^2 + \frac{\mu_o}{\gamma^2 M}\left(\frac{\alpha}{\gamma}\right)\omega_1\omega_2(\omega_1-\omega_2)^2$$

$$\theta = \tan^{-1}\frac{\frac{\mu_o}{\gamma^2 M}\left(\frac{\alpha}{\gamma}\right)\omega_1\omega_2(\omega_1-\omega_2)}{\omega_1^2\omega_2^2\left(\frac{\mu_o}{\gamma^2 M}\right)^2 + \omega_1\omega_2\left(\frac{\alpha}{\gamma}\right)^2 - H_p^2} .$$

If

$$\omega_1^2\omega_2^2 \left(\frac{\mu_o}{\gamma^2 M}\right)^2 >> \omega_1\omega_2 \left(\frac{\alpha}{\gamma}\right)^2$$

$$\rho \approx \omega_1^2\omega_2^2 \left(\frac{\mu_o}{\gamma^2 M}\right)^2 - H_p^2 \qquad 55$$

$$\theta = \frac{\frac{\alpha}{\gamma}(\omega_1-\omega_2)}{\omega_1\omega_2\frac{\mu_o}{\gamma^2 M}} . \qquad 56$$

If ω_1 and ω_2 are nearly equal, then the angle θ will be very nearly equal to zero. Therefore oscillation and amplification are possible. The threshold pump field is

$$H_p = \sqrt{\frac{1}{Q_1 Q_2}} \frac{V_{cavity}}{V_{sample}} \frac{\mu_o}{M} \left(\omega_1^2 \frac{\mu_o}{\gamma^2 M} \omega_2^2 \frac{\mu_o}{\gamma^2 M} - H_p^2\right). \qquad 57$$

By comparing these results with equation 51, it is seen that this threshold pump field is larger than in the low-frequency case, because of the fact that

$$\omega_1^2 \frac{\mu_o}{\gamma^2 M} > H_k \qquad \omega_2^2 \frac{\mu_o}{\gamma^2 M} > H_k.$$

However, as the frequency is increased, the cavity size is greatly reduced, therefore the filling factor will be improved. But, at the same time, the Q of the cavity will also be reduced since the Q of a cavity is inversely proportional to the square root of the frequency. A numerical example follows. Let

$$\omega_1 = 2\pi \times 10^9 \text{ cps.}$$

If

$$\omega_2 = 1.5\omega_1$$

$$\rho e^j \approx 1.85 \times 10^6 + j.3 \times 10^6$$

$$\theta \approx 9^o$$

$$\frac{V_{cavity}}{V_{film}} = 1.5 \times 10^6$$

then

$$\sqrt{Q_1 Q_2} \approx 10^3$$

$$H_{ps} \approx 3500 \text{ ampere-turn/meter.}$$

IV. PHASE RELATION BETWEEN THE SIGNAL AND IDLING FIELDS

If one takes the conjugate of equation 44, one obtains the following pair of equations:

$$\left(\frac{\omega_1}{Q_1} + 2\lambda\right)A_1 = j\omega_1 \frac{H_pM}{\mu_o \rho} e^{-j\theta} F_1 A_2^* \qquad 58$$

$$\left(\frac{\omega_2}{Q_2} + 2\lambda\right)A_2 = j\omega_2 \frac{H_pM}{\mu_o \rho} e^{j\theta} F_2 A_1^*. \qquad 59$$

By setting

$$\lambda = 0$$

$$A_1 = K_1 e^{j\psi_1}$$

$$A_2 = K_2 e^{j\psi_2}$$

one finds that

$$\frac{1}{Q_1}\frac{1}{Q_2} K_1K_1 \text{Cos}(\psi_1+\psi_2) = -\left(\frac{H_pM}{\mu_o\rho}\right)^2 F_1F_2 \text{Cos}(\psi_1+\psi_2)K_1K_2$$

$$\frac{1}{Q_1}\frac{1}{Q_2} K_1K_2 \text{Sin}(\psi_1+\psi_2) = \left(\frac{H_pM}{\mu_o\rho}\right)^2 F_1F_2 \text{Sin}(\psi_1+\psi_2)K_1K_2.$$

Using the threshold pump field relation, one gets

$$\text{Cos}(\psi_1+\psi_2) = -\text{Cos}(\psi_1+\psi_2) \qquad 60$$

$$\text{Sin}(\psi_1+\psi_2) = \text{Sin}(\psi_1+\psi_2). \qquad 61$$

In other words, this is really saying that

$$\psi_1 + \psi_2 = \frac{n}{2}\pi\,(n = \pm 1, \pm 3, ----). \qquad 62$$

Note that it was assumed that H_p, the pump field, has zero phase. In other words H_p was used as the reference.

Therefore ψ_1 and ψ_2 are the phase angles of signal field and idling field with respect to the pump field. Equation 62 merely states that the phase angle between the signal field and the idling field has a definite relation. Indeed, this is true in all two-tank-circuit parametric amplifiers. This phase relation is set up automatically, if the device has an idling circuit along with a signal circuit.

Now suppose that one has only a single circuit, then

$$2\psi = \psi_1 + \psi_2 \qquad A_1 = A_2.$$

From equation 62, one has

$$2\psi = \frac{n}{2}\pi \quad (n = \pm 1, \pm 3 \text{ ----}). \qquad 63$$

In other words, in order for oscillation to build up, the phase relation between the signal and the pump has to satisfy a certain condition. This phase locking property has been used in a magnetic film parametron (9).

V. GAIN, BANDWIDTH, AND EFFECTIVE Q OF THE AMPLIFIER

In the preceding sections the oscillator was discussed. Now it is possible to proceed to the case of an amplifier. In doing so the same procedure will be followed as in section III. However, the driving function to the cavity will be a signal field. Assume that the signal field is

$$H_s = Be^{j\omega_1 t}. \qquad 64$$

Then equation 37 becomes

$$\ddot{a}_1 + \Omega_1^2 a_1 = -\frac{d^2}{dt^2}\frac{1}{\mu_o}\frac{\int_s M_1 h_1 dv}{\int_v h_1^2 dv} + \frac{d}{dt}\frac{B}{\mu_o}e^{j\omega_1 t}. \qquad 65$$

To find the steady-state solution, assume that

$$a_1 = A_1 e^{j\omega_1 t}. \qquad 66$$

Then by the same procedure as in section III, one finds

$$\left[-\omega_1^2 + \Omega_1^2 + j\frac{\omega_1^2}{Q_1}\right]A_1 = -\frac{\omega_1^2}{\mu_o}\frac{MH_p}{\rho e^{j\theta}}F_1 A_2^* + j\omega_1\frac{B}{\mu_o}.$$

Let

$$\omega_1 = \Omega_1 + \delta$$

then

$$\left[-2\delta\omega_1 + j\frac{\omega_1^2}{Q_1}\right]A_1 = -\frac{\omega_1^2}{\mu_o}\frac{MH_p}{\rho e^{j\theta}}F_1 A_2^* + j\omega_1\frac{B}{\mu_o}.$$

Similarly

$$\left(j\frac{\omega_2^2}{Q_2} - 2\delta\omega_2\right)A_2^* = -\frac{\omega_2^2}{\mu_o}\frac{MH_p}{\rho e^{j\theta}}F_2 A_1.$$

Then

$$(\frac{\omega_1}{Q_1} + 2j\delta)A_1 = j\frac{\omega_1}{\mu_o}\frac{MH_pF_1}{\rho e^{j\theta}}A_2^* - \frac{B}{\mu_o} \qquad 67$$

$$(\frac{\omega_2}{Q_2} + 2j\delta)A_2^* = -j\frac{\omega_1}{\mu_o}\frac{MH_pF_2}{\rho e^{j\theta}}A_1 \qquad 68$$

$$(\frac{\omega_2}{Q_2} + 2j\delta)(\frac{\omega_1}{Q_1} + 2j\delta)A_1$$

$$= \omega_1\omega_2\left[\frac{H_pM}{\mu_o\rho e^{j\theta}}\right]^2 F_1F_2A_1 - (\frac{\omega_2}{Q_2} + 2j\delta)\frac{B}{\mu_o}\,. \qquad 69$$

If it is assumed that $\theta \approx 0$ and

$$\left[\frac{H_pM}{\mu_o\rho}\right]^2 F_1F_2 = H_F$$

then

$$A_1 = \frac{-(\frac{\omega_2}{Q_2} + 2j\delta)\frac{B}{\mu_o}}{\frac{\omega_2}{Q_2}\frac{\omega_1}{Q_1} + 2\delta j\,(\frac{\omega_2}{Q_2} + \frac{\omega_1}{Q_1}) - 4\delta^2 - \omega_1\omega_2H_F}$$

$$= \frac{-\frac{B}{\mu_o}}{\left[\frac{\omega_1}{Q_1} - \frac{\omega_1\omega_2H_F}{\frac{\omega_1}{\omega_2} + 2j\delta}\right] + 2\delta j}$$

$$= \frac{-\frac{B}{\mu_o}}{\left[\frac{\omega_1}{Q_1} - \frac{\omega_1\omega_2^2H_F/Q_2}{(\frac{\omega_2}{Q_2})^2 + 4\delta^2}\right. + 2\delta j\left[1 + \frac{\omega_1\omega_2H_F}{(\frac{\omega_2}{Q_2})^2 + 4\delta^2}\right]}\,. \qquad 70$$

The bandwidth is defined as the frequency difference between the half-power points; therefore, from equation 70, one obtains the bandwidth for the signal as

$$\Delta\omega = \frac{\dfrac{\omega_1}{Q_1} - \dfrac{\omega_1\omega_2^2 H_F/Q_2}{\left(\frac{\omega_2}{Q_2}\right)^2 + 4\delta^2}}{1 + \dfrac{\omega_1\omega_2 H_F}{\left(\frac{\omega_2}{Q_2}\right)^2 + 4\delta^2}} \quad . \qquad 71$$

When the pump field is zero, the bandwidth is

$$\Delta\omega = \frac{\omega_1}{Q_1} \; .$$

Comparing this with equation 71, one finds the effective Q for the case when the pump field is present. The value of Q_{eff} is given by

$$\frac{1}{Q_{eff}} = \frac{\dfrac{1}{Q_1} - \dfrac{\omega_2 H_F/Q_2}{\left(\frac{\omega_2}{Q_2}\right)^2 + 4\delta^2}}{1 + \dfrac{\omega_1\omega_2 H_F}{\left(\frac{\omega_2}{Q_2}\right)^2 + 4\delta^2}} \quad . \qquad 72$$

If

$$\frac{\omega_2}{Q_2} >> \delta$$

$$\Delta\omega = \frac{\omega_1\left(\frac{1}{Q_1} - Q_2 H_F\right)}{1 + \dfrac{Q_2^2 H_F}{\omega_2}} \qquad 73$$

$$\frac{1}{Q_{eff}} = \frac{\dfrac{1}{Q_1} - Q_2 H_F}{1 + \dfrac{Q_2^2 H_F}{\omega_2}} \quad . \qquad 74$$

In the case of an oscillator with the pump field at its threshold value, one can replace H_F by $\frac{1}{Q_1}\frac{1}{Q_2}$, then one finds

$$Q_2 H_F = \frac{1}{Q_1}$$

and

$$\frac{1}{Q_{eff}} = 0 \qquad Q_{eff} = \infty .$$

This is indeed the condition for self-oscillation. Therefore, if H_p is below the oscillating threshold then

$$Q_2 H_F < \frac{1}{Q_1} .$$

The $\frac{Q_2^2 H_F}{\omega_2}$ varies according to the value of H_p. It is zero at zero pump field and it becomes 1 at threshold, provided one assumes that $\frac{\omega_1}{Q_1}$ is about equal to $\frac{\omega_2}{Q_2}$. Therefore the denominator of equation 74 varies from 1 to 2. Now set

$$\frac{1}{Q_{eff}} = \frac{1}{\beta}\left(\frac{1}{Q_1} - \frac{1}{Q_s}\right) \tag{75}$$

where

$$\frac{1}{Q_s} = Q_2 H_F \qquad 0 \leq \frac{1}{Q_s} \leq \frac{1}{Q_1} \tag{76}$$

$$\beta = 1 + \frac{Q_2^2 H_F}{\omega_2} \qquad 1 \leq \beta \leq 2. \tag{77}$$

Note also that Q_1 is the loaded Q of the cavity, which can be expressed as

$$\frac{1}{Q_1} = \frac{1}{Q_a} + \frac{1}{Q_e}$$

where Q_a is the unloaded Q of the cavity and Q_e is the external Q which related the load to the cavity. If one draws the equivalent circuit of the cavity at resonance, one obtains the following two circuits, one for the case before the pump is applied, which is shown in Figure 2, and another for the case after the pump is applied, which is shown in Figure 3. In Figure 3

$$\frac{1}{Q} = \frac{1}{Q_{eff}} - \frac{1}{Q_1}$$

$$= -\left[\frac{\frac{1}{\beta}}{Q_s} + \frac{1 - \frac{1}{\beta}}{Q_1}\right]$$

which indicates a negative conductance element. Therefore the insertion power gain is

$$PG_{in} = \left[\frac{\frac{1}{Q_a} + \frac{1}{Q_e}}{\frac{1}{Q_a} + \frac{1}{Q_e} - \frac{1}{Q}}\right]^2$$

$$= \frac{Q_{eff}^2}{Q_1^2} \quad . \qquad 78$$

Equation 78 checks closely with the results of **A. A.** Read (3) if one makes the following manipulation:

$$PG_{in} = \left[\frac{\frac{1}{Q_{eff}}}{\frac{1}{Q_1}}\right]^2$$

$$= \left[1 - \frac{\frac{1}{Q_1} - \frac{1}{\beta Q_1} + \frac{Q_2 H_F}{\beta}}{\frac{1}{Q_1}}\right]^2 .$$

If H_p is very close to the threshold

$$H_F \approx \frac{1}{Q_1}\frac{1}{Q_2} \approx \frac{1}{Q_2^2}$$

$$\beta \approx 2.$$

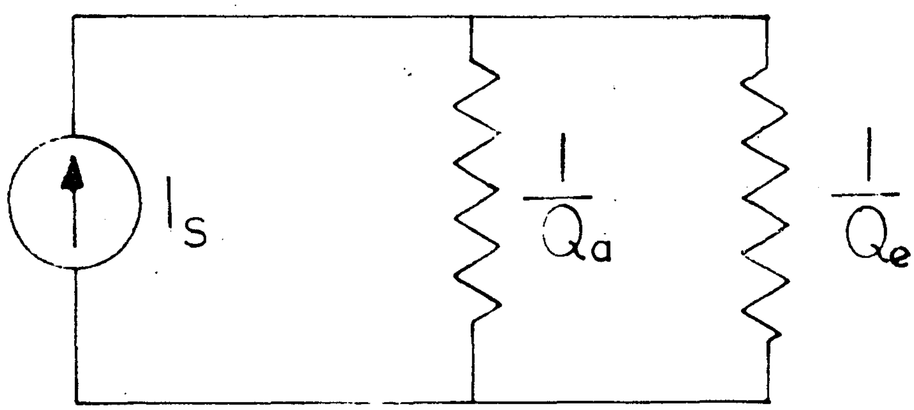

Figure 2. Equivalent circuit of a cavity at resonance before the pump field is applied

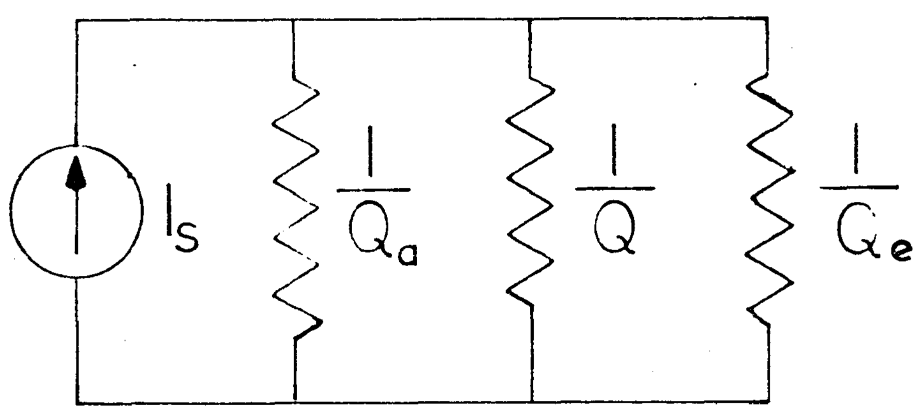

Figure 3. Equivalent circuit of a cavity at resonance after the pump field is applied

Then

$$PG_{in} = \left[1 - \frac{\frac{1}{Q_2}}{\frac{1}{Q_1}}\right]^2$$

and

$$Q_1 = \frac{2\pi f_1 w_1}{I_1^2 R_1}$$

$$Q_2 = \frac{2\pi f_2 w_2}{I_2^2 R_2}$$

where w_1 and w_2 are the energys stored in the cavity modes 1 and 2. Also, if $w_1 = w_2$, then

$$PG_{in} = \left[1 - \frac{I_2^2 R_2/f_2}{I_1^2 R_1/f_1}\right]^2 .$$

This is Read's result.

VI. GENERALIZED POWER RELATION

In general one can write a vector identity

$$\nabla\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot\nabla\times\vec{E} - \vec{E}\cdot\nabla\times\vec{H}. \qquad 79$$

From Maxwell's equation, one has

$$\nabla\times\vec{E} = -\frac{\delta}{\delta t}(\mu_o\vec{H}+\vec{M}) \qquad 80$$

$$\nabla\times\vec{H} = \frac{\delta\vec{D}}{\delta t} + \sigma\vec{E}. \qquad 81$$

Therefore, from equation 79

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\vec{H}\cdot(\mu_o\frac{\delta\vec{H}}{\delta t} + \frac{\delta\vec{M}}{\delta t}) - \vec{E}\cdot(\frac{\delta\vec{D}}{\delta t} + \sigma\vec{E}). \qquad 82$$

Taking the volume integral of equation 82 around the magnetic material sample and using the divergence theorem, one finds

$$-\int_{sample}\vec{E}\times\vec{H}ds = \int_{sample}\vec{H}\cdot\frac{d\vec{M}}{dt}dv + \mu_o\int_{sample}\vec{H}\cdot\frac{\delta\vec{H}}{\delta t}dv$$
$$+\int_{sample}\vec{E}\cdot(\frac{\delta\vec{D}}{\delta t} + \sigma\vec{E})dv. \qquad 83$$

The left side of the above equation is the surface integral of Poynting's vector around the magnetic sample. The negative sign means that the energy flows into the sample. If one neglects other effects and looks only for the power flow due to the change of the magnetization in the sample, one finds that

$$P_{out} = -\int_s \frac{d\vec{M}}{dt}\cdot\vec{H}dv.$$

The average power that the sample puts out due to the change of magnetization is

$$Pav = -\frac{1}{2}\,\mathrm{Re}\int_s \frac{d\vec{M}}{dt}\cdot\vec{H}^{*}dv. \qquad 84$$

Using equations 28 and 29, one finds the power coupled out to be

$$P_1(\omega_1) = -\frac{1}{2}\,\mathrm{Re}\,j\omega_1 \frac{M|A_1|^2 \rho_2 e^{-j\theta_2}}{\rho e^{j\theta}} \int_s h_{t1}^2 dv$$

$$+\frac{1}{2}\,\mathrm{Re}\frac{j\omega_1\ MH_p A_2^{*} A_1^{*}}{\rho e^{j\theta}} \int_s h_{t1}h_{t2}dv$$

$$P_2(\omega_2) = -\frac{1}{2}\,\mathrm{Re}\frac{-j\omega_2 M|A_2|^2 \rho_1 e^{j\theta_1}}{\rho e^{j\theta}} \int_s h_{t2}^2 dv$$

$$+\frac{1}{2}\,\mathrm{Re}\frac{-j\omega_2 MH_p A_2 A_1}{\rho e^{j\theta}} \int_s h_{t1}h_{t2}dv.$$

The first term on the right side of both of the above equations represents the losses of the magnetic material itself. The second term is the power coupled out by the pump field. By taking account of both positive and negative frequency components, one finds that the power output is

$$P_1(\omega_1) = \mathrm{Re}\frac{j\omega_1 MH_p A_1^{*} A_2^{*}}{\rho e^{j\theta}} \int_s h_{t1}h_{t2}dv \qquad 85$$

$$P_2(\omega_2) = \mathrm{Re}\frac{-j\omega_2 MH_p A_2 A_1}{\rho e^{j\theta}} \int_s h_{t1}h_{t2}dv. \qquad 86$$

Set

$$A_1 = |A_1|\, e^{j\psi_1}$$

$$A_2 = |A_2|\, e^{j\psi_2}\ .$$

Then equations 85 and 86 become

$$P_1(\omega_1) = \frac{\omega_1 M}{\rho} H_p |A_1| \, |A_2| \, [\mathrm{Sin}(\psi_1+\psi_2)\mathrm{Cos}\,\theta_1$$
$$+ \mathrm{Sin}\,\theta_1 \mathrm{Cos}(\psi_1+\psi_2)] \int_s h_{t1} h_{t2} dv \qquad 87$$

$$P_2(\omega_2) = \frac{\omega_2 M}{\rho} H_p |A_1| \, |A_2| \, [\mathrm{Sin}(\psi_1+\psi_2)\mathrm{Cos}\,\theta_1$$
$$- \mathrm{Sin}\,\theta_1 \mathrm{Cos}(\psi_1+\psi_2)] \int_s h_{t1} h_{t2} dv. \qquad 88$$

As shown in section IV for a two-tank circuit, the phase relation is $\psi_1+\psi_2 = \frac{\pi}{2}$. Therefore the net power flow is

$$P_1(\omega_1) = \frac{\omega_1 M}{\rho} H_p |A_1| \, |A_2| \, \mathrm{Cos}\,\theta_1 \int_s h_{t1} h_{t2} dv \qquad 89$$

$$P_2(\omega_2) = \frac{\omega_2 M}{\rho} H_p |A_1| \, |A_2| \, \mathrm{Cos}\,\theta_1 \int_s h_{t1} h_{t2} dv. \qquad 90$$

Taking the ratio of the above equations, one obtains

$$\frac{P_1(\omega_1)}{P_2(\omega_2)} = \frac{\omega_1}{\omega_2} \qquad 91$$

which is the Manley-Rowe relation.

Now to generalize the result which was found in the case of a cavity, one would assume that a system uses magnetic film to couple power from a pump into that system. To look for the oscillatory case, one would assume that the H-field in that system is building up with time. In other words, one would replace

$$A_1 h_1 \quad \text{by} \quad A_1 h_1 e^{\lambda t} \qquad 92$$

and

$$A_2 h_2 \quad \text{by} \quad A_2 h_2 e^{\lambda t}. \qquad 93$$

As this H-field is increasing with time the total energy stored should also be increased. The pump must furnish this amount of energy and also the loss of the system through the coupling of the magnetic film. One then equates these two powers and seeks a solution such that λ is zero or positive real.

By the theory of conservation of energy, the power that a system absorbs is

$$P_a = \frac{dw}{dt} + \omega_1 \frac{w}{Q_1} \tag{94}$$

where W is the energy stored in the system, ω_1 is the oscillating frequency, and Q_1 is the Q of the system.

In general, the total energy stored in a system can be represented by

$$W = \mu o \int_v A_1 e^{\lambda t} A_1^* e^{\lambda t} h_1^2 dv \tag{95}$$

$$\frac{dw}{dt} = 2\lambda \mu o \left|A_1\right|^2 \int_v h_1^2 dv \quad e^{2\lambda t} \tag{96}$$

$$\frac{\omega_1 w}{Q_1} = \frac{\omega_1 \mu o \left|A_1\right|^2 \int_v h_1^2 dv}{Q_1} \quad e^{2\lambda t}. \tag{97}$$

If there is no other power source in the system one would have

$$(2\lambda + \frac{\omega_1}{Q_1}) \mu o \left|A_1\right|^2 \int_v h_1 dv$$

$$= (\omega_1 - j\lambda) \frac{M}{\rho} H_p \left|A_1\right| \left|A_2\right| \cos\theta_1 \int_s h_{t1} h_{t2} dv.$$

By neglecting λ at the right side of this equation, and by similar processes for the idling circuit, one finds the following pair of equations

$$(2\lambda + \frac{\omega_1}{Q_1}) |A_1|^2 = \omega_1 \frac{MH_p}{\rho\mu_o} |A_1| |A_2| \cos\theta \frac{\int_s h_{t1} h_{t2} dv}{\int_v h_1^2 dv} \quad 98$$

$$(2\lambda + \frac{\omega_2}{Q_2}) |A_2|^2 = \omega_2 \frac{MH_p}{\rho\mu_o} |A_1| |A_2| \cos\theta \frac{\int_s h_{t1} h_{t2} dv}{\int_v h_1^2 dv} \quad 99$$

or

$$(2\lambda + \frac{\omega_1}{Q_1})(2\lambda + \frac{\omega_2}{Q_2}) = \omega_1\omega_2\left[\frac{MH_p}{\rho\mu_o}\right]^2 F_1F_2(\frac{1}{2} + \frac{1}{2}\cos 2\theta).$$

For

$$\lambda \geqq 0$$

$$\frac{1}{Q_1}\frac{1}{Q_2} \leq (\frac{MH_p}{\rho\mu_o})^2 F_1F_2(\frac{1}{2} + \frac{1}{2}\cos 2\theta). \quad 100$$

In equations 85 and 86 only the real part of the power is used. However the presence of the magnetic film also introduces additional inductance in the system which will change the resonant frequency of the system. Hence the λ as assumed in equations 92 and 93 should be complex. The power output of the magnetic film due to the pump should have both active and reactive parts. Hence, if one equates the active power delivered to the active power absorbed, and the reactive power delivered to the reactive power absorbed (or the imaginary part of equation 94, one finds that the threshold pump field condition will be exactly the same as in section III. This idea of reactive power and changing of resonant frequency of the system will be used in section VII for the parametron case.

VII. PARAMETRON

If the oscillator discussed previously has only one tank circuit, then the oscillating signal remains in a certain phase relation with the pump field, as was pointed out in section III. This single tank circuit or subharmonic oscillator is called a parametron. To examine its threshold oscillating condition, an approach similar to sections III and VI can be used.

By using equation 84, one finds

$$P = -\frac{1}{2}\int_{sample} \frac{d\vec{M}_T}{dt} \cdot \vec{H}_T^* dv$$

$$M_T = M\phi_1 e^{j\omega_1 t}$$

$$H_T^* = (H_k - \frac{\mu_o}{\gamma^2 M}\omega_1^2 - j\omega_1 \frac{\alpha}{\gamma})\phi_1^* + H_p\phi_1.$$

The above two equations are defined in section II. Therefore

$$P = -\frac{1}{2}\int_{sample}\left[j\omega_1 M\phi_1\phi_1^*(H_k - \frac{\mu_o}{\gamma^2 M}\omega_1^2 - j\omega_1\frac{\alpha}{\gamma}) + j\omega_1 M H_p\phi_1\phi_1\right]dv.$$

Since the frequency has both negative and positive components, the power, therefore, should be double this value. The integration is carried throughout the volume of the magnetic film sample. If one assumes that all fields and magnetization are uniform inside the sample, one finds

$$P = -[j\omega_1 M\phi_1\phi_1^*(H_k - \frac{\mu_0}{\gamma^2 M}\omega_1^2 - j\omega_1\frac{\alpha}{\gamma})V_f + j\omega_1 MH_p\phi_1\phi_1 V_f$$

where V_f is volume of the magnetic material.

If ω_1 is far below the resonant frequency of the magnetic material, one can assume that

$$H_k >> \frac{\mu_0}{\gamma^2 M}\omega_1^2.$$

The phase relation of ϕ can be expressed as

$$\phi_1 = \phi_m(\cos\theta + j\sin\theta)$$

where ϕ_m is the absolute magnitude of ϕ_1 and θ is the phase angle of ϕ with respect to the pump field. One then finds

$$P = -j\omega_1\phi_m^2(H_k - j\omega_1\frac{\alpha}{\gamma})V_f - j\omega_1 MH_p\phi_m^2 V_f(\cos 2\theta + j\sin 2\theta).$$

This power has two components, namely the active component and the reactive component.

$$P_{active} = -\omega_1^2 M\phi_m^2\frac{\alpha}{\gamma}V_f + \omega_1 MH_p\phi_m^2\sin 2\theta V_f \qquad 101$$

$$P_{reactive} = -\omega_1\phi_m^2 MH_k V_f - \omega_1 MH_p\phi_m^2\cos 2\theta V_f \qquad 102$$

If one assumes that a small coil was wound around the magnetic material, whose hard direction was parallel to the axis of the coil, and if the magnetic material had a thickness T, width W, and length ℓ, then the induced voltage in this small coil would be

$$V = \frac{d\lambda}{dt}$$

where λ is the flux linkage of this coil. According to the above assumption

$$\lambda = MTWN\phi_1 e^{j\omega_1 t}$$

$$V = j\omega_1 MWTN\phi e^{j\omega_1 t}$$

or

$$V^2 = (\omega_1 MWNT\phi_m)^2.$$

If this is substituted into equations 101 and 102, one finds

$$P_{active} = -\frac{V^2\frac{\alpha}{\gamma}}{\frac{MN^2TW}{\ell}} + \frac{H_p V^2}{\omega_1\frac{MN^2TW}{\ell}}\sin 2\theta \qquad 103$$

$$P_{reactive} = -\frac{H_k V^2}{\omega_1\frac{MN^2TW}{\ell}} - \frac{H_p V^2}{\omega_1\frac{MN^2TW}{\ell}}\cos 2\theta. \qquad 104$$

Now, defining

$$L_o = \frac{MN^2TW}{H_a \ell}$$

where H_a is the anisotropy field, then

$$P_{active} = -\frac{V_1^2\frac{\alpha}{\gamma}}{H_a L_o} + \frac{H_p V^2}{H_a\omega_1 L_o}\sin 2\theta \qquad 105$$

$$P_{reactive} = -\frac{H_k V^2}{H_a L_o \omega_1} - \frac{H_p V^2}{H_a \omega_1 L_o}\cos 2\theta. \qquad 106$$

The last term of the right side of equation 105 represents a positive active power term. This term can be interpreted either as a negative resistance or as a power source. This is the term which couples the pump energy into the oscillator circuit.

If the coil is connected to an RLC series circuit, one can then set up the equivalent circuit shown in Figure 4.

In Figure 4

$$R_f = \frac{\gamma H_a L_o}{\alpha}$$

$$L_f = \frac{H_a L_o}{H_k}$$

L_a = The external inductance which includes the air inductance of the coil and other lead inductance

C = The tank circuit capacitance

R = The resistance of the coil and the external lead resistance.

If the circuit of Figure 4 is in resonance, then

$$\frac{H_p V^2}{H_a \omega_1 L_o} \operatorname{Sin} 2\theta = \frac{V^2 \frac{\alpha}{\gamma}}{H_a L_o} + \frac{R_a V^2}{R_a^2 + (\omega_1 L_a - \frac{1}{\omega_1 C})}$$

$$- \frac{H_p V^2}{H_a \ 1 L_o} \operatorname{Cos} 2\theta = \frac{V_2 H_k}{H_a L_o \ 1} + \frac{V_2(\omega_1 L_a - \frac{1}{\omega_1 C})}{R_a^2 + (\omega_1 L_a - \frac{1}{\omega_1 C})^2}$$

or

$$\frac{H_p}{H_a} \operatorname{Sin} 2\theta = \frac{1}{H_a} + \omega_1 L_o \frac{R_a}{R_a^2 + (\omega_1 L_a - \frac{1}{\omega_1 C})^2} \qquad 107$$

$$- \frac{H_p}{H_a} \operatorname{Cos} 2\theta = 1 + \frac{H_b}{H_a} + \frac{\omega_1 L_a - \frac{1}{\omega_1 C}}{R_a^2 + (\omega_1 L_a - \frac{1}{\omega_1 C})^2} \ . \qquad 108$$

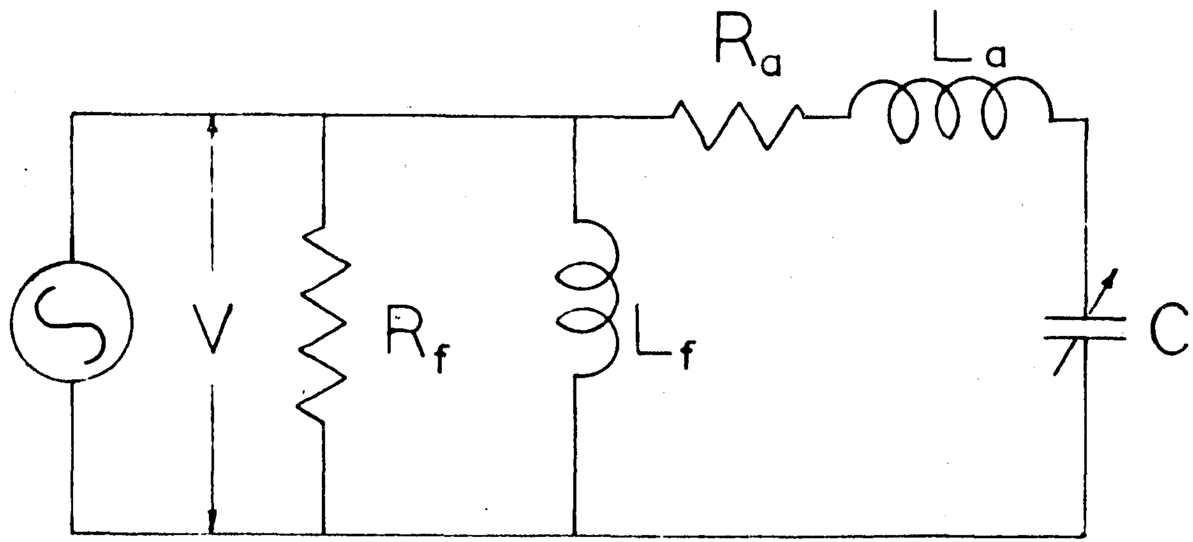

Figure 4. Equivalent circuit of magnetic thin film parametron

These two equations should describe the threshold pump field in relation to different circuit parameters to a good degree of accuracy.

VIII. EXPERIMENTAL RESULTS

In order to observe any appreciable effects on the cavity-type oscillator and amplifier as described in sections III and IV, a strong pump field is needed. To generate this field in a cavity, a rather high power r-f oscillator is required. The available r-f oscillator does not put out enough power for this purpose. Therefore no experimental work has been done on this cavity-type oscillator.

A magnetic-film type of parametron was built and has been working very fine in a range of pump frequencies from 200 mc. to 360 mc. However, most of the data was taken at 230 mc. and 260 mc.

The nonlinear inductor of the parametron was made of two pieces of thin film of approximately 80 percent Ni and 20 percent Fe which were vacuum plated on glass substrates, each having a thickness of six mils. The detailed fabrication of this inductor is shown in Figures 5(a) and 5(b). The magnetic film had a thickness of about 2000Å to 3000Å, while the width was 1.25 mm. and length 2.5 mm. A center conductor, made of one mil thick copper strip and having 1.25 mm. width was placed between these two film pieces. The configuration was then a sandwich of glass, film, copper, film and glass. A small coil of 11 turns of No. 40, A.W.G. wire was then wound very tightly around the films with its axis along the hard direction of the film. The center conductor on one end was then

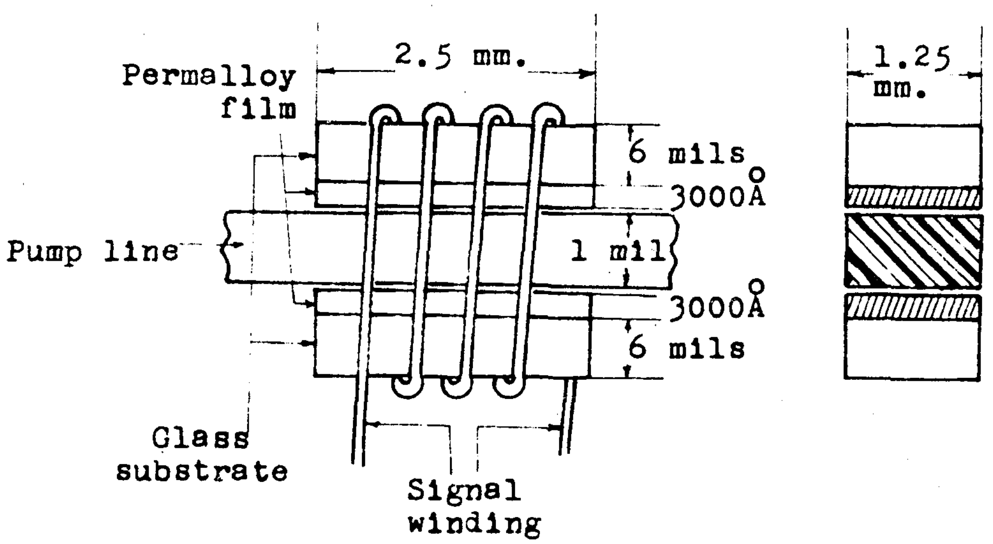

Figure 5(a). Construction diagram of magnetic film inductor

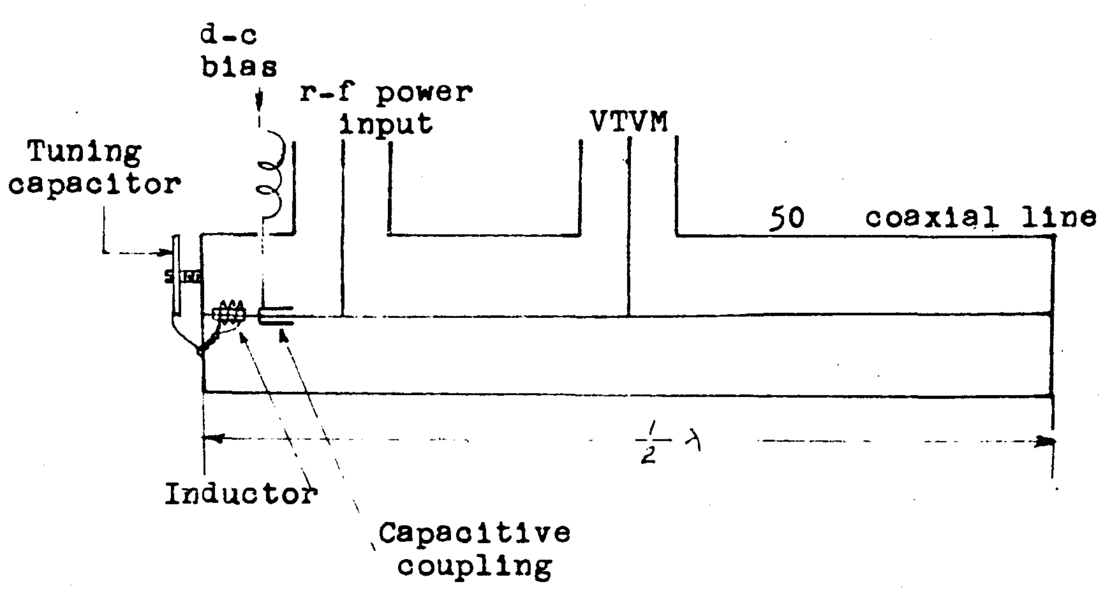

Figure 5(b). Construction diagram of magnetic thin film parametron connected to the shorted end of a 50 ohm coaxial line, while the other end was capacitively coupled to the center conductor of the coaxial line. Two very short leads brought the signal coil terminals through a small hole at the shorted end of the coaxial line which were then connected to a variable capacitor. The pump field was fed into the coaxial line through a coaxial tee. A Hewlett-Packard 410B high frequency VTVM was connected through another tee at approximately the center of the coaxial resonant pump line. This tee is specially made for coaxial-line voltage measurement. An adjustable shorted stub was placed at the other end of the coaxial pump line, so that the pump circuit was essentially a half-wave resonant tank shorted at both ends. The d-c bias was applied to the center copper strip of the inductor through an r-f choke coil which had about four to five turns of A.W.G. No. 37 wire, so that the r-f power would not leak away. A Textronix 585 high-frequency oscilloscope was used to detect the oscillator's signal.

The inductance of the magnetic-film inductor was measured by a Boonton 250-A R-X meter. Since the film sample showed multiple domains at zero bias and the inductance reading fluctuated, a bias of about 200 ampere-turns/meter was applied in the easy direction which gave the maximum inductance reading. At 114.5 mc. this inductance was measured to be 16 x 10^{-8} Henry. Then a very strong field supplied by a horseshoe permanent magnet was applied in the easy direction, thereby clamping the magnetization, and the air inductance was then read which was 9.7×10^{-8} Henry at 114.5 mc. The difference of these two measurements will give the maximum inductance of the film or L_o in equations 107 and 108, which is about 6.3×10^{-8} Henry. If one uses the definition of L_o and calculates this inductance, one finds that

$$L_o = \frac{MTWN^2}{(H_a+H_b)\ell} \;.$$

By taking

$N = 11$

$T = 2 \times 2.5 \times 10^{-7}$ meter

$W = 1.25$ mm.

$= 2.5$ mm.

$H_a = 300$ ampere-turns/meter

$H_b = 200$ ampere-turns/meter

then $L_o \approx 6.1 \times 10^{-8}$ Henry, which is very close to the value actually measured in the bridge.

By combining equations 107 and 108, one obtains the following equation

$$\left(\frac{H_p}{H_a}\right)^2 = \left[\omega_1 \frac{\alpha}{\gamma H_a} + \omega_1 L_o \frac{R_a}{R_a^2+(\omega_1 L_a - \frac{1}{\omega_1 C})^2}\right]^2$$

$$+ \left[1 + \frac{H_p}{H_a} + \frac{(\omega_1 L - \frac{1}{\omega_1 C})\,\omega_1 L_o}{R_a^2+(\omega_1 L_a - \frac{1}{\omega_1 C})^2}\right]^2 \;. \qquad 109$$

In this equation H_a is the anisotropy field plus the small d-c bias field which used to hold the film in a single domain. It has a value of approximately 500 ampere-turns/meter.

C is the external tuning capacitor which can be measured by loosely coupling a signal into the parametron circuit with the pump power off, and tuning it to resonance at a certain frequency. With known values of L_a and L_o this capacitance can be very easily calculated.

R_a is the external loss. In this experiment it was mainly due to the resistance of the inductance coil. Its value is judged to be 0.5 ohm based on calculation. However, from the measurement of the Q of the tank circuit, the total series resistance is approximately seven ohms. Therefore, one can assume that most of the loss is due to magnetic film, and to simplify the analysis it is assumed that this R is negligible.

From equation 105, the loss of the film is equal to $\frac{\alpha V^2}{\gamma H_a L_o}$. If the external loss is negligible, then the Q of the tank circuit can be represented as

$$\frac{1}{Q} = \frac{\alpha \omega_1 (L+L_o)}{\gamma H_a L_o} .$$

This Q has been measured to be approximately 10 at 114.5 mc. If one takes this value of Q, and the value of L_o and L_a with the assumption that $\gamma = 2.25 \times 10^5$, one can solve for α and finds it to be 0.0388. From this α one can compute the $\omega_1 \frac{\alpha}{\gamma H_a}$ term in equation 109 to be approximately 0.25. This was found to be about half the value of the experimental result. This discrepancy might be explained by the fact that as the parametron oscillates at half the pump frequency it also generates higher harmonics which will capacitively couple back into the pump line. This causes a loss in the signal tank circuit. This additional loss does not show in equation 109. The magnitude of this loss was approximately determined by measuring the Q of the tank circuit with the pump line connected and with the pump line physically disconnected. The Q of the signal tank circuit changed from 10 to 5 in these two measurements. It is apparent that the pump line introduces additional losses in the system. To account for this empirically determined loss, the best value of $\frac{\omega_1 \alpha}{\gamma H_a}$ was found to be around 0.5.

From equation 109 a theoretical curve of threshold pump field versus the d-c bias field can be plotted, and it agrees very closely with the experimental result. This is shown in Figure 6. The disagreement at lower bias values is considered to be due to domain break-up of the film.

The saturation amplitude of the oscillator can not be predicted from this analysis, because the ϕ angle is far from small at saturation. However, it can be approximately computed by assuming that the ϕ angle is near 90 degrees. The induced voltage computed by this assumption is about 4.85 volts. Using an oscilloscope, one observes a voltage of about 3.8 volts. However, since the oscillator's frequency was above the bandwidth of the scope, considerable attenuation ex-

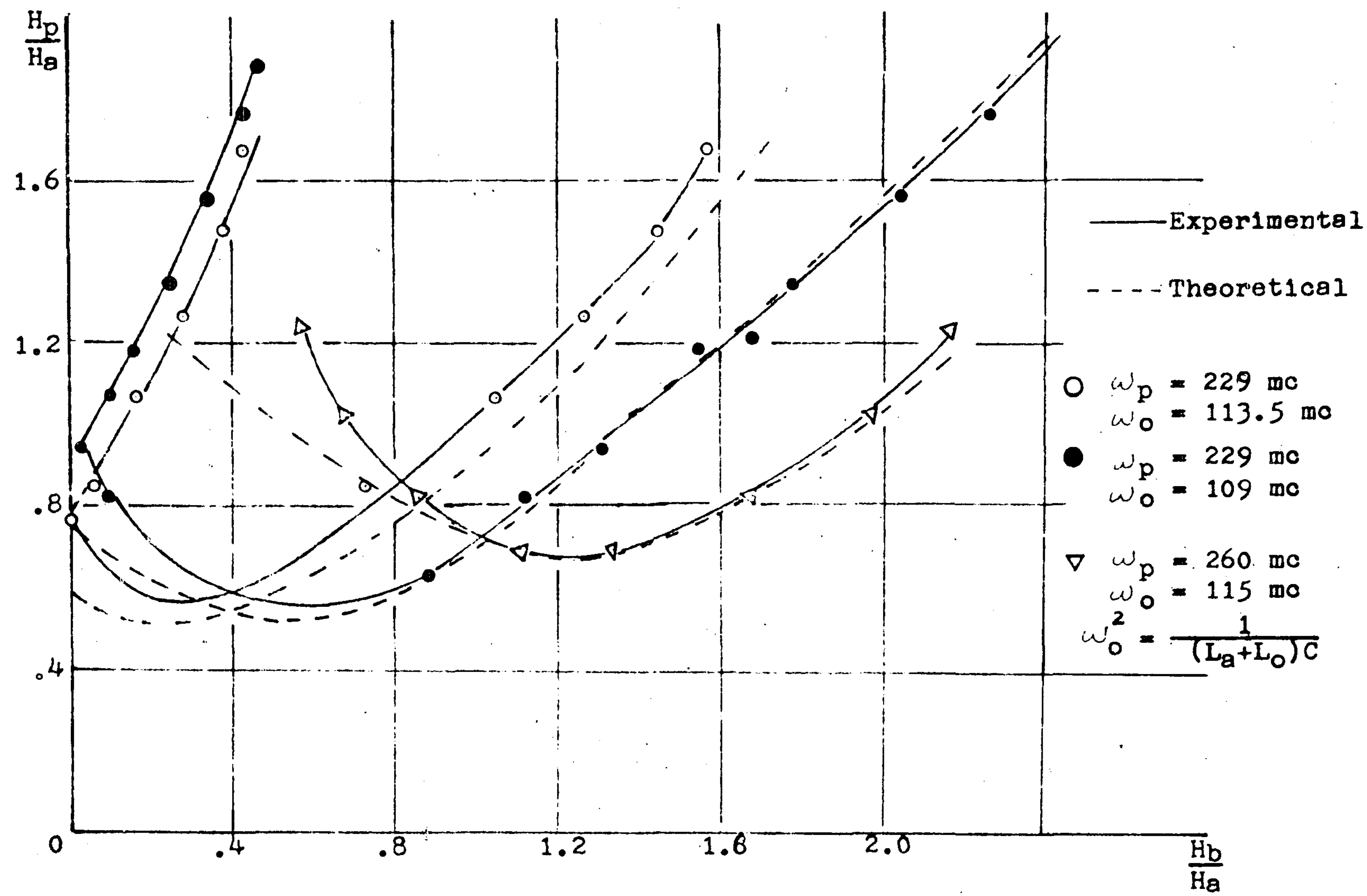

Figure 6. Plot of threshold pump field versus bias field

isted, hence the observed voltage would be smaller than the actual voltage.

The pump line input power ranged approximately from 0.5 watts to 1.5 watts. The Q of the pump line was very low, about 20. If the Q of the pump line were improved, the input power could be greatly reduced. The power that the parametron took seemed to be extremely low. The available equipment does not have enough sensitivity to detect the power level difference when the parametron is in oscillation or not in oscillation.

IX. SUMMARY

The purpose of this investigation was to study in analytical detail the possibility of use of thin, single-domain ferromagnetic film for parametric amplifiers and oscillators at very high frequency. The ferromagnetic film was made of 80 Ni and 20 Fe which was vacuum evaporated on a glass substrate. This thin film can be used as a nonlinear element which will couple pump power into the signal circuit. If the pump field is larger than a certain threshold value, free oscillation will occur. Hence, for an oscillator this threshold pump field is a very important parameter.

The first part of this study was to use a resonant cavity as both signal and idling circuit. Also this cavity will support the pump field mode. The magnetic thin film was oriented in the cavity in such a way that it will couple the pump power into the signal and idling modes. By use of the small amplitude general motion equation of the magnetization of magnetic film, the threshold pump field for oscillation was found. A discussion of the phase angle of this signal field and the operating frequency range was given. Later, a signal input is applied in this cavity, it then will act as an amplifier. The gain, bandwidth, and effective Q of the signal mode was given in terms of the pump field.

In the last part of this thesis, a single tank circuit oscillator (or subharmonic generator) was discussed. Experimental work demonstrated that this oscillator worked very fine at a pump frequency from 200 to 360 mc. The theoretical prediction and the experimental result of the plot of pump field versus the d-c bias field agrees very well.

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